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Asymmetric-unbalanced counterflow thermal regenerator problem: solution by the Galerkin method and meaning of dimensionless parameters

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Abstract—The asymmetric-unbalanced counterflow thermal regenerator problem described by the classical idealizations is solved by the Galerkin method. The integral equations relating to the reversal conditions at cyclic equilibrium of the regenerator matrix are transformed into a set of algebraic equations. This permits the determination of the expansion coefficients associated with the representation of the matrix temperature distributions at the start of each period of the cycle in the form of a power series in terms of the space variable. The method is easy and straightforward to apply and leads to explicit analytical expressions for the expansion coefficients for any combination of the four dimensionless parameters of the asymmetric-unbalanced regenerator. Excellent agreement has been found between the results of this new solution and those reported in the literature for different numerical solutions. Convergence towards the exact results by computations to higher order terms is discussed. The solution has been used to predict the effectiveness of a wide range of the four dimensionless parameters. Thermodynamic reasons for an alternative but rational and meaningful way of defining the four regenerator parameters are presented.

INTRODUCTION

IT WAS in 1972 when Hausen [1], surveying the theories of heat transfer in regenerators, wrote:

> "... as far as I know, the question to what degree all these methods connected with the integral equation are suitable to practical problems, is not yet answered sufficiently ..."

It is the intention of this paper to contribute to answering Hausen's question.

The simplest mathematical representation of fixed bed cyclic thermal regenerators has remained virtually static since the initial publication of Nusselt [2], and likewise, the rotary matrix exchanger has also stayed in the same state since the original work of Coppage and London [3]. In both systems, the sole mechanism of heat transfer between the flowing gases and the regenerator matrix is assumed to be forced convection, and this results in two coupled first-order partial differential equations describing the energy transfer in each of two periods of operation. Despite the simplicity of the differential equations under classical assumptions, their solution has proved to be challenging, and performances of counterflow regenerator have been widely investigated numerically as well as analytically. The state-of-the-art and survey texts on this subject appeared in the books by Hausen [4] and Schmidt and Willmott [5], and the work of Razelos [6]. Available closed methods for solving the counterflow regenerator problem are mainly related to the governing differential equations. The method of lines was used by Hill and Willmott [7] in order to reduce the counterflow regenerator problem to a set of ordinary differential equations in time. Their approach uses the trapezoidal rule to discretize one of the governing partial differential equations, and is related to the method proposed by Razelos [8]. Recently the computation speed of this method has been improved by Hill and Willmot [9]. However, all these methods avoid the use of the integral equations. As stated at the beginning, this paper is a presentation of the method for solving the counterflow regenerator problem formulated by the integral equations. It overcomes the difficulties of the method of Iliffe [10] and Nahavandi and Weinstein [11], both used for solving the integral equations, by looking for a solution in a class of special functions that identically satisfy the governing differential equations.

For design purposes a regenerator is usally considered to have attained cyclic equilibrium, i.e. the fluids and matrix temperature distributions are repeated in successive cycles. The solution of the governing differential equations is presented in terms of the regenerator effectiveness as a function of pertinent dimensionless groups. The specific form of these dimensionless groups is to some extent optional, and the two most common forms are: the number of transfer units–capacity rate ratio method (generally used for rotary regenerators) whereby

$$\varepsilon = \varepsilon(N_{\mathrm{tu},0}, C^*, C_{\mathrm{R}}^*, (\alpha A)^*) \tag{1}$$

and the reduced length–reduced period method (generally used for fixed-matrix regenerators) in which

NOMENCLATURE

A matrix to gas heat transfer area [m^2]
A_{mk} constants defined by equation (87)
a_{1m} expansion coefficients in trial solution, equation (65)
a_{2m} expansion coefficients in trial solution, equation (66)
B_{mk} constants defined by equation (88)
C^* unbalance factor, β [dimensionless]
C_k constants defined by equation (85)
C_R^* reciprocal of minimal utilization factor, $1/U_1$ [dimensionless]
c_p specific heat of gas at constant pressure [J kg^{-1} K^{-1}]
c_s specific heat of solid matrix [J kg^{-1} K^{-1}]
$\mathbb{F}_1(\xi)$ matrix temperature distribution at the start of period 1 [dimensionless]
$\mathbb{F}_2(\zeta)$ matrix temperature distribution at the start of period 2 [dimensionless]
$F_i(y, z)$ special function defined by equation (37)
$G_i(y, z)$ special function defined by equation (38)
$I_n(\cdot)$ modified Bessel function of nth (integer) order
i, j, k, l, m, n integers (counters)
L regenerator length in the flow direction [m]
$\mathbb{L}_1\{\mathbb{F}_1, \mathbb{F}_2\}$ integral equation operator, defined by equation (63)
$\mathbb{L}_2\{\mathbb{F}_1, \mathbb{F}_2\}$ integral equation operator, defined by equation (64)
$\mathscr{L}$ Laplace transform operator
M order of the trial solution
$\dot{M}$ gas mass flow rate [kg s^{-1}]
M_s total mass of solid matrix [kg]
$N_{tu,0}$ modified number of transfer units, $\Lambda_1/(1+\sigma\beta)$ [dimensionless]
P duration of gas flow period [s]
p Laplace transform variable ($\eta \rightarrow p$)
Q_{act} actual quantity of heat being transferred in the regenerator [J]
Q_{max} maximal quantity of heat that can be transferred in the regenerator, defined by equation (26) [J]
$\mathscr{R}$ residuals of the integral equations, defined by equations (79) and (80)
Re (s) real part of complex variable s
s Laplace transform variable ($\xi \rightarrow s$)
T temperature [K]
t time variable [s]
U utilization factor, $\Pi/\Lambda = (\dot{M}c_pP)/(M_sc_s)$ [dimensionless]
u dummy variable
$V_i(y, z)$ special functions defined by equation (35)
$V_{i,0}(y, z)$ special functions defined by equation (36)
v dummy variable
x coordinate along the fluid 1 flow direction [m]
y dummy variable
z dummy variable.

Greek symbols
α gas to matrix heat transfer coefficient [W m^{-2} K^{-1}]
$(\alpha A)^*$ reduced period ratio, $\Pi_1/\Pi_2 = \sigma\beta$ [dimensionless]
β unbalance factor, $U_1/U_2 = (\dot{M}c_pP)_1/(\dot{M}c_pP)_2$ [dimensionless]
ε regenerator effectiveness, defined by equation (27) [dimensionless]
ζ complementary spatial coordinate, $1-\xi$ [dimensionless]
η dimensionless time variable, t/P [dimensionless]
θ dimensionless temperature, $(T-T_{2,in})/(T_{1,in}-T_{2,in})$ [dimensionless]
Λ reduced length for a regenerator, $\alpha A/(\dot{M}c_p)$ [dimensionless]
ξ dimensionless spatial coordinate, x/L [dimensionless]
Π reduced period for a regenerator, $\alpha AP/(M_sc_s)$ [dimensionless]
σ asymmetry factor, Λ_1/Λ_2 [dimensionless].

Subscripts
c cold
h hot
in inlet
m matrix
out outlet
s1 solid in period 1
s2 solid in period 2
1 smaller U
2 larger U.

Superscripts
$\tilde{}$ approximation
$\bar{}$ Laplace transform.

$$\varepsilon = \varepsilon(\Lambda_h, \Lambda_c, \Pi_h, \Pi_c). \quad (2)$$

These two representations are equivalent as was shown by Shah [12], while Heggs [13] detailed the rotary system parameters in terms of fixed bed ones. The compilation of the relationships between the dimensionless groups of the two methods is given by Shah [14]. This one-to-one correspondence between the two methodologies allows the results obtained either in the form of equation (1) or (2) to be used for both types of regenerators.

The problem of establishing the $\varepsilon = \varepsilon(\Lambda_h, \Lambda_c, \Pi_h, \Pi_c)$ relationship is in fact that of evaluating the actual heat transfer rate at cyclic equilibrium. Since no closed-form solutions of the mathematical model of counterflow thermal regenerator problem are available, and since four different dimensionless groups are the parameters of any solution, all previous attempts to solve the problem, either numerically or analytically, have been predestined to a rather limiting range of parameters. Evidently there is a need for: (i) a reliable and readily applicable method for solving the counterflow regenerator problem in a wide range of all governing parameters, and (ii) a rational way of presenting the results.

Recently it was demonstrated that the Galerkin method yields a solution that predicts very accurately the regenerator effectiveness of symmetric-balanced systems with $1 \leqslant \Lambda \leqslant 2000$ and $0 \leqslant \Pi \leqslant 4000$ [15]. This paper is intended as a proof that the adoption of the Galerkin method for solving the general counterflow regenerator problem is a sufficient step in supplementing the first of the above needs. The asymmetric-unbalanced regenerator problem was first attacked via the Galerkin method [16]. Latterly it has been shown that the Successive Integral Method when applied to the same problem, but with no reference to the integral equations, is equivalent to the Galerkin method [17]. The presentation of the complete set of results for the range of four regenerator parameters of practical interest is given in ref. [18]. Here, only the sample results for a randomly selected set of four regenerator parameters are presented together with the thermodynamic reasons for an alternative but rational way of defining the regenerator parameters.

THE DIFFERENTIAL EQUATIONS AND REGENERATOR PARAMETERS

The classical thermal regenerator model [1, 2] is based on the following idealizations:

(a) total thermal capacitance of the solid regenerator matrix is constant;

(b) heat capacity rates of both fluids are constant;

(c) thermal conductances for convective transfer between the fluid and matrix are uniform and constant;

(d) transit times required for a gas particle to flow through the regenerator are negligibly small compared to the gas flow periods;

(e) solid matrix material offers no resistance to heat flow in the direction normal to fluid flow;

(f) no heat is conducted axially.

Based on these assumptions, an energy balance provides two equations applicable during the hot gas flow period

$$\frac{1}{\Lambda_h}\frac{\partial T_h}{\partial \xi} = -\frac{1}{\Pi_h}\frac{\partial T_m}{\partial \eta} = T_m - T_h \quad (3)$$

and two equations during the cold gas flow period

$$\pm\frac{1}{\Lambda_c}\frac{\partial T_c}{\partial \xi} = -\frac{1}{\Pi_c}\frac{\partial T_m}{\partial \eta} = T_m - T_c. \quad (4)$$

The regenerator can operate with either unidirectional (plus sign in equation (4)) or counterflow of streams (minus sign in equation (4)). Since the mathematical model for the unidirectional operation mode has been solved by exact analytical procedures [19], only the counterflow regenerator problem will be discussed in this paper.

In equations (3) and (4) ξ is the fractional distance along the flow path in the regenerator matrix of length L, and η is the fractional completion of a respective gas flow period. Temperatures of the hotter gas, of the colder gas and of the solid matrix are denoted by T_h, T_c and T_m, respectively. The four parameters Λ_h, Λ_c (reduced lengths), Π_h and Π_c (reduced periods) are defined in the Nomenclature.

Differential equations (3) and (4) describe the regenerator operation when the appropriate boundary conditions are specified. These are constant gas inlet temperature $T_{h,in}$ and $T_{c,in}$ at the opposite ends ($\xi = 0$ and 1) of the regenerator matrix, and the condition stating that the matrix temperature field at the end of one gas flow period is the initial matrix temperature distribution for the subsequent gas flow period.

In this paper the mathematical model of the counterflow thermal regenerator is considered in dimensionless form in the space and time domain as shown in Fig. 1. As seen from Fig. 1 the space domain $0 \leqslant \xi = x/L \leqslant 1$ is unique for both periods, while the time domains $0 \leqslant \eta_1 = t/P_1 \leqslant 1$ and $0 \leqslant \eta_2 = t/P_2 \leqslant 1$ are separate for each period. The gas flows, i.e. the flow periods, are not distinguished by the attributes 'hotter' or 'colder', but by 'weaker' and 'stronger'. The subscript 1 is assigned to the weaker gas flow period such that the respective $U_1 = (\Pi/\Lambda)_1$ ratio is the smaller of the two ratios: $U_h = (\Pi/\Lambda)_h$ and $U_c = (\Pi/\Lambda)_c$. The use of the Π/Λ ratio as a regenerator parameter instead of reduced period Π was first suggested by Johnson [20] who termed it the 'utilization factor'. The main advantage of using

$$U_h = (\Pi/\Lambda)_h \quad \text{and} \quad U_c = (\Pi/\Lambda)_c \quad (5)$$

is the fact that these parameters do not contain the fluid to matrix heat transfer coefficients of the respective flow periods. Thus, defining

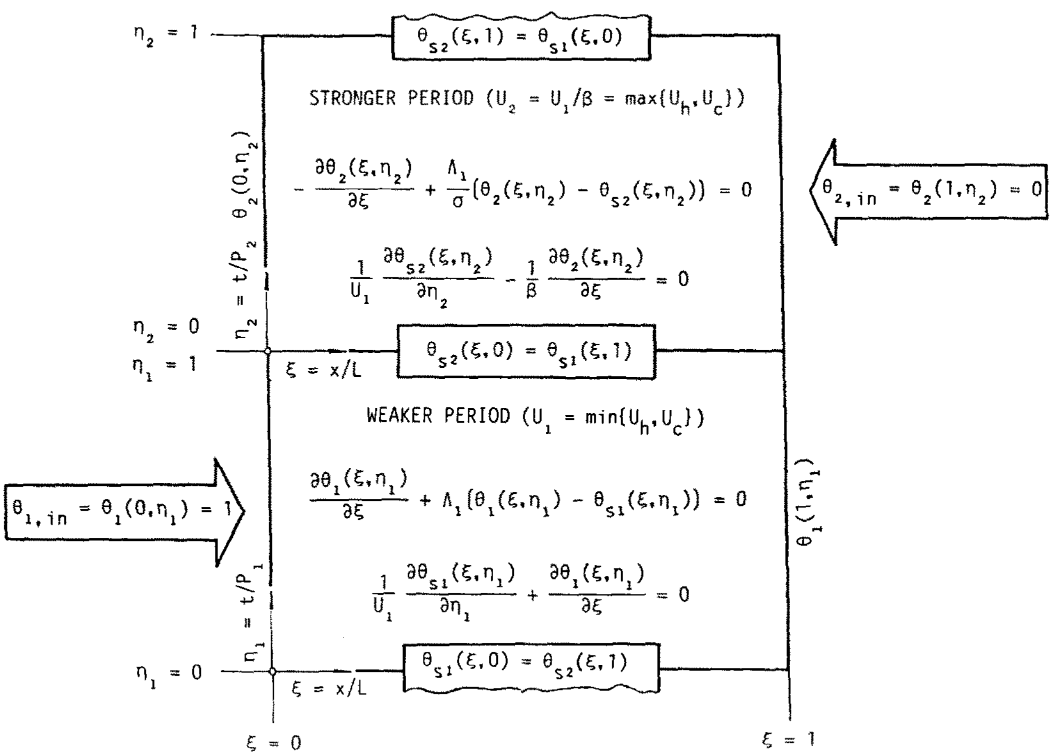

FIG. 1. Space ($0 \leqslant \xi \leqslant 1$) and time ($0 \leqslant \eta_1, \eta_2 \leqslant 1$) domain of a counterflow regenerator in cyclic equilibrium.

$$U_1 = \min\{U_h, U_c\} \tag{6}$$

the matrix and fluid enthalpy balance for the weaker period can be written in dimensionless form as

$$\frac{1}{U_1}\frac{\partial\theta_{s1}(\xi,\eta_1)}{\partial\eta_1} + \frac{\partial\theta_1(\xi,\eta_1)}{\partial\xi} = 0 \tag{7}$$

in $0 \leqslant \eta_1 \leqslant 1$ and $0 \leqslant \xi \leqslant 1$. The physical meaning of U_1 can be seen from

$$U_1 = \frac{\int_0^1 [\theta_{s1}(\xi,1) - \theta_{s1}(\xi,0)]\,d\xi}{1-\theta_{1,out}} = \frac{\frac{1}{L}\int_0^L [T_{s1}(x,P_1) - T_{s1}(x,0)]\,dx}{T_{1,in} - T_{1,out}} = \left.\frac{\text{mean matrix temperature change}}{\text{mean gas temperature change}}\right\} \text{ in the weaker period} \tag{8}$$

where the gas 1 outlet temperature (see Fig. 1) is

$$\theta_{1,out} = \int_0^1 \theta_1(1,\eta_1)\,d\eta_1. \tag{9}$$

From equation (8) it is clear what is meant by terming U the utilization factor. The word stands for utilizing the regenerator matrix as an intermediator in transferring heat from one to the other gas. The goal is not to make a large mean matrix temperature change in this process, and obviously for a favorable regenerator operation U_1 should not be greater than unity, but on the contrary much less than unity.

The utilization factor in the stronger period, where subscript 2 is assigned to all quantities, is

$$U_2 = \max\{U_h, U_c\} \tag{10}$$

and the matrix and fluid enthalpy balance equation for counterflow operation of a regenerator is

$$\frac{1}{U_1}\frac{\partial\theta_{s2}(\xi,\eta_2)}{\partial\eta_2} - \frac{1}{\beta}\frac{\partial\theta_2(\xi,\eta_2)}{\partial\xi} = 0 \tag{11}$$

in $0 \leqslant \eta_2 \leqslant 1$ and $0 \leqslant \xi \leqslant 1$, where

$$\beta = U_1/U_2 \leqslant 1 \tag{12}$$

is introduced to eliminate U_2. Schmidt and Willmott [5] termed β the unbalance factor. The main advantage of using the unbalance factor is the overall enthalpy balance for the regenerator can be written in the form equivalent to any two-fluid heat exchanger

$$\theta_{2,out} = \beta(1-\theta_{1,out}) \tag{13}$$

where the outlet fluid temperatures are for the regenerator given by equation (9) and

$$\theta_{2,out} = \int_0^1 \theta_2(0,\eta_2)\,d\eta_2. \tag{14}$$

The physical meaning of the unbalance factor can be seen from

$$\beta = \frac{\theta_{2,\mathrm{out}}}{1-\theta_{1,\mathrm{out}}} = \frac{T_{2,\mathrm{out}}-T_{2,\mathrm{in}}}{T_{1,\mathrm{in}}-T_{1,\mathrm{out}}}$$

$$= \frac{\text{total gas temperature change in the stronger period}}{\text{total gas temperature change in the weaker period}}. \tag{15}$$

Hence, as for any two-fluid heat exchanger, the higher the unbalance ($\beta \ll 1$) the better the regenerator performance (higher effectiveness).

As shown in Fig. 1 the heat transfer equation for the weaker period

$$\frac{\partial \theta_1(\xi,\eta_1)}{\partial \xi} + \Lambda_1[\theta_1(\xi,\eta_1)-\theta_{s1}(\xi,\eta_1)] = 0 \tag{16}$$

introduces the reduced length Λ_1 (dimensionless heat transfer coefficient) as the third regenerator parameter. Its physical meaning is

$$\Lambda_1 = \frac{1-\theta_{1,\mathrm{out}}}{\int_0^1\int_0^1 [\theta_1(\xi,\eta_1)-\theta_{s1}(\xi,\eta_1)]\,\mathrm{d}\xi\,\mathrm{d}\eta_1}$$

$$= \frac{T_{1,\mathrm{in}}-T_{1,\mathrm{out}}}{\frac{1}{LP_1}\int_0^{P_1}\int_0^L [T_1(x,t)-T_{s1}(x,t)]\,\mathrm{d}x\,\mathrm{d}t}$$

$$= \left.\frac{\text{total gas temperature change}}{\text{average transfer potential}}\right\} \text{ in the weaker period} \tag{17}$$

so that it plays a role of the Stanton number for the regenerator. Thermodynamically speaking it is desirable to produce a given gas temperature change at as low as possible average transfer potential. This would be a more reversible heat transfer process in the sense of lower entropy production. Thus, for a thermodynamically favorable operation of a regenerator it is desirable to have high values of Λ_1.

Finally, the fourth differential equation, describing heat transfer in the stronger period (see Fig. 1)

$$-\frac{\partial \theta_2(\xi,\eta_2)}{\partial \xi} + \frac{\Lambda_1}{\sigma}[\theta_2(\xi,\eta_2)-\theta_{s2}(\xi,\eta_2)] = 0 \tag{18}$$

introduces the fourth dimensionless parameter

$$\sigma = \Lambda_1/\Lambda_2 \tag{19}$$

the asymmetry factor. It is the ratio of relative average transfer potentials in the stronger and weaker period:

$$\sigma = \frac{\dfrac{\int_0^1\int_0^1 [\theta_{s2}(\xi,\eta_2)-\theta_2(\xi,\eta_2)]\,\mathrm{d}\xi\,\mathrm{d}\eta_2}{\theta_{2,\mathrm{out}}}}{\dfrac{\int_0^1\int_0^1 [\theta_1(\xi,\eta_1)-\theta_{s1}(\xi,\eta_1)]\,\mathrm{d}\xi\,\mathrm{d}\eta_1}{1-\theta_{1,\mathrm{out}}}}$$

$$= \frac{\dfrac{\frac{1}{LP_2}\int_0^{P_2}\int_0^L [T_{s2}(x,t)-T_2(x,t)]\,\mathrm{d}x\,\mathrm{d}t}{T_{2,\mathrm{out}}-T_{2,\mathrm{in}}}}{\dfrac{\frac{1}{LP_1}\int_0^{P_1}\int_0^L [T_1(x,t)-T_{s1}(x,t)]\,\mathrm{d}x\,\mathrm{d}t}{T_{1,\mathrm{in}}-T_{1,\mathrm{out}}}}$$

$$= \frac{\left.\dfrac{\text{average transfer potential}}{\text{total gas temperature change}}\right\}\text{ in the stronger period}}{\left.\dfrac{\text{average transfer potential}}{\text{total gas temperature change}}\right\}\text{ in the weaker period}}. \tag{20}$$

As stated above Λ_1 should be as high as possible for the favorable regenerator operation. Then, from the definition of σ, Λ_2 should be at least of the same order of magnitude or higher. Thermodynamics, thus, imposes the study of $\sigma \leqslant 1$ for properly designed regenerators.

Differential equations (7), (11), (16) and (18) together with the following boundary conditions:

$$\theta_1(0,\eta_1) = 1 \tag{21}$$

$$\theta_2(1,\eta_2) = 0 \tag{22}$$

$$\theta_{s1}(\xi,0) = \theta_{s2}(\xi,\eta_2 = 1) \tag{23}$$

$$\theta_{s2}(\xi,0) = \theta_{s1}(\xi,\eta_1 = 1) \tag{24}$$

describe the operation of a counterflow thermal regenerator as a function of four parameters: U_1, Λ_1, β and σ. The classical parameters discussed in the introduction are readily established from the following set of relations: $\Pi_1 = U_1\Lambda_1$, $\Pi_2 = U_1\Lambda_1/\beta\sigma$, $N_{\mathrm{tu},0} = \Lambda_1/(1+\sigma\beta)$, $C^* = \beta$, $C_R^* = 1/U_1$, $(\alpha A)^* = \sigma\beta$.

The main advantage of introducing these four independent dimensionless parameters (U_1, Λ_1, β and σ) of the regenerator operation is that one can easily identify the physical meaning of some special cases corresponding to the limiting values of these parameters. One can deduce the following operating conditions:

(1) $U_1 \to 0$ corresponds to the aperiodic (or 'recuperative') operation of a regenerator. The case when the matrix temperature distribution is *time independent* and is the same in both periods. Fluid temperatures are in the steady state as well.

(2) $U_1 \to \infty$ means no process at all and the exchanger ceases to exist. This case is not to be considered.

(3) $\Lambda_1 \to 0$ corresponds to 'short' regenerators—a case with no heat transfer at all—not of practical interest.

(4) $\Lambda_1 \to \infty$ corresponds to 'long' regenerators—a case with vanishing transfer potential in both periods.

(5) $\beta \to 0$—a completely unbalanced exchanger.

The case with no temperature change throughout the stronger period for any value of σ. Thermodynamically the best operation for given U_1 and Λ_1.

(6) $\beta \to 1$—a balanced regenerator. Each stream undergoes exactly the same overall temperature change.

(7) $\sigma \to 0$—a completely asymmetric regenerator. The case when there is no transfer potential in the stronger period. This case together with $U_1 \to 0$ and $\Lambda_1 \to \infty$ yields the highest effectiveness for any given unbalance factor β.

(8) $\sigma \to 1$—symmetric regenerator.

(9) $\sigma \to \infty$ designates a disappearance of the stronger period implying that there is no process at all. This case should not be considered. Actually, even the case with $\sigma > 1$ should not be studied since the regenerator performance deteriorates with increasing values of σ.

Real interest is limited to just six of the above special cases ($U_1 \to 0$; $\Lambda_1 \to \infty$; $\beta \to 0$; $\beta \to 1$; $\sigma \to 0$ and $\sigma \to 1$) and they must be studied separately. This paper addresses the general unbalanced and asymmetric case, i.e. $1 > \beta > 0$ and $1 > \sigma > 0$.

REGENERATOR EFFECTIVENESS

The effectiveness of any two-fluid heat exchanger essentially is a dimensionless and normalized measure of the quantity of heat actually being transferred between the two streams. The normalization requires the recognition of the maximum possible fluid enthalpy change in the system. This hypothetical quantity of heat (Q_{max}) can be seen as the enthalpy change of the weak stream undergoing the maximum possible temperature change ($T_{h,in} - T_{c,in}$). A 'weak stream', or more precisely a 'weaker period' of a regenerator is the one with the smaller of the two possible heat capacities, $(\dot{M}c_pP)_h$ and $(\dot{M}c_pP)_c$. Thus, the attribution of 'min' should be given to the period for which the relation

$$(\dot{M}c_pP)_{min} = \min\,[(\dot{M}c_pP)_h, (\dot{M}c_pP)_c] \quad (25)$$

holds, so that

$$Q_{max} = (\dot{M}c_pP)_{min}(T_{h,in} - T_{c,in}). \quad (26)$$

The regenerator effectiveness is then simply defined as

$$\varepsilon = Q_{act}/Q_{max} \quad (27)$$

and it is a unique measure of its thermal performance. By the uniqueness here it is meant that the same ε should be obtained by writing Q_{act} either in terms of hot period parameters or in terms of cold period parameters. This was demonstrated [21] for a model of the diabatic regenerator, and the results given there reduce, for the adiabatic model under consideration, to

$$\varepsilon = (\dot{M}c_pP)_h(T_{h,in} - T_{h,out})/(\dot{M}c_pP)_{min}(T_{h,in} - T_{c,in})$$
$$= (\dot{M}c_pP)_c(T_{c,out} - T_{c,in})/(\dot{M}c_pP)_{min}(T_{h,in} - T_{c,in}). \quad (28)$$

In order to rewrite these equations in terms of the regenerator parameters used in this paper (utilization factors $U_1 = (\Pi/\Lambda)_{min}$ and $U_2 = (\Pi/\Lambda)_{max}$, and the appropriate dimensionless temperatures) divide both the denominators and numerators by the matrix total heat capacity $M_s c_s$, and assign correspondingly the subscripts 1 and 2. This will yield

$$\varepsilon = \frac{T_{1,in} - T_{1,out}}{T_{1,in} - T_{2,in}} = 1 - \theta_{1,out} \quad (29)$$

or

$$\varepsilon = \frac{U_2}{U_1}\frac{T_{2,out} - T_{2,in}}{T_{1,in} - T_{2,in}} = \frac{1}{\beta}\theta_{2,out} \quad (30)$$

where $\theta_{1,out}$ is the mean outlet gas temperature evaluated, using equation (9), from the gas outlet temperature distribution $\theta_1(1,\eta_1)$ in the weaker period. Similarly $\theta_{2,out}$ is the mean outlet gas temperature evaluated, using equation (14), from the gas outlet temperature distribution $\theta_2(0,\eta_2)$ in the stronger period. $\theta_{1,out}$ and $\theta_{2,out}$ are the results of the solution of the mathematical model and will, thus, depend on four regenerator parameters. This will yield a relationship

$$\varepsilon = \varepsilon(U_1, \Lambda_1, \beta, \sigma) \quad (31)$$

which can be expressed also in terms of the classical parameters as in equation (1) or (2).

Having the complete solution to the problem one can evaluate the regenerator effectiveness from any of the two equations (29) and (30), but it should be emphasized that there are three more equivalent expressions. These arise from alternative ways of expressing Q_{act}, and are as follows.

From the average driving force (transfer potential) in the weaker period

$$\varepsilon = \Lambda_1 \int_0^1 \int_0^1 [\theta_1(\xi,\eta_1) - \theta_{s1}(\xi,\eta_1)]\,d\xi\,d\eta_1. \quad (32)$$

From the average driving force (transfer potential) in the stronger period

$$\varepsilon = \frac{\Lambda_1}{\beta\sigma} \int_0^1 \int_0^1 [\theta_{s2}(\xi,\eta_2) - \theta_2(\xi,\eta_2)]\,d\xi\,d\eta_2. \quad (33)$$

From the energy accumulated in the matrix during one period

$$\varepsilon = \frac{1}{U_1} \int_0^1 [\theta_{s1}(\xi,\eta_1 = 1) - \theta_{s2}(\xi,\eta_2 = 1)]\,d\xi. \quad (34)$$

As stated above ε can be computed from any of the alternative formulae, but it appears that, for the method presented in this paper, equation (34) is the most convenient.

FUNCTION USED FOR THE SOLUTION

For a concise notation a class of special functions of two variables is extensively used throughout this paper. These are defined by the following Laplace transform pairs:

$$\mathscr{L}^{-1}_{p\to z}\left\{\frac{\exp[-yp/(p+1)]}{p^i}\right\} = V_i(y,z)$$

$$\equiv \exp(-y-z)\sum_{n=i-1}^{\infty}\binom{n}{i-1}(z/y)^{n/2}I_n(2\sqrt{(yz)}) \quad (35)$$

$$\mathscr{L}^{-1}_{p\to z}\left\{\frac{\exp[-yp/(p+1)]}{(p+1)^i}\right\} = V_{i,0}(y,z)$$

$$\equiv \exp(-y-z)(z/y)^{(i-1)/2}I_{i-1}(2\sqrt{(yz)}) \quad (36)$$

for $i = 1,2,3,\ldots, y,z \geq 0$. The notation V_i and $V_{i,0}$ is due to Serov and Korol'kov [22], but these functions are also related to the families of functions $F_i(y,z)$ and $G_i(y,z)$ introduced by Romie [23]. Note that Romie's [23] functions are

$$F_i(y,z) = V_{i+1}(y,z) \quad (37)$$

$$G_i(y,z) = (-1)^{i+1}\left[V_{1,0}(y,z)+\sum_{n=1}^{i+1}(-1)^n V_n(y,z)\right] \quad (38)$$

for $i = 0, 1, 2, \ldots,$ and

$$F_{-1}(y,z) = V_{2,0}(y,z) \quad (39)$$

$$G_{-1}(y,z) = V_{1,0}(y,z). \quad (40)$$

Some of the properties of V_i and $V_{i,0}$ functions that are of importance in this paper are

$$V_1(y,0) = \exp(-y), \quad V_1(0,z) = 1 \quad (41)$$

$$V_i(y,0) = 0, \quad V_i(0,z) = \frac{z^{i-1}}{(i-1)!}, \quad i = 2,3,4,\ldots \quad (42)$$

$$\mathscr{L}_{y\to p}\{V_{2,0}(y,z)\} = \exp[-yp/(p+1)]-\exp(-y) \quad (43)$$

$$1-V_1(y,z) = V_1(z,y)-V_{1,0}(y,z) \quad (44)$$

$$V_{1,0}(y,z) = V_{1,0}(z,y) \quad (45)$$

$$V_{2,0}(y,z) = \frac{y}{z}V_{2,0}(z,y). \quad (46)$$

For $n = 0,1,2,\ldots$ the following integrals hold:

$$\int_0^y \frac{\xi^n}{n!}V_{1,0}(y-\xi,z)\,\mathrm{d}\xi = G_n(y,z) \quad (47)$$

$$\int_0^y \frac{\xi^n}{n!}V_{2,0}(y-\xi,z)\,\mathrm{d}\xi = V_{n+1}(z,y)-\exp(-z)\frac{y^n}{n!} \quad (48)$$

$$\int_0^z \frac{\eta^n}{n!}V_i(y,\eta)\,\mathrm{d}\eta = \sum_{m=0}^{n}(-1)^m\frac{z^{n-m}}{(n-m)!}V_{i+m+1}(y,z). \quad (49)$$

Easy ways to compute these functions are described in refs. [15, 23].

INTEGRAL EQUATIONS

The *formal* solution to the equations of the weaker period, equations (7) and (16), with the inlet fluid temperature given by equation (21) and *arbitrary* initial matrix temperature distribution

$$\theta_{s1}(\xi,0) = \mathbb{F}_1(\xi) \quad (50)$$

is given by

$$\theta_1(\xi,\eta_1) = V_1(\Lambda_1\xi, U_1\Lambda_1\eta_1) + \Lambda_1\int_1^{\xi}\mathbb{F}_1(u)V_{1,0}[\Lambda_1(\xi-u), U_1\Lambda_1\eta_1]\,\mathrm{d}u \quad (51)$$

and

$$\theta_{s1}(\xi,\eta_1) = V_1(\Lambda_1\xi, U_1\Lambda_1\eta_1) - V_{1,0}(\Lambda_1\xi, U_1\Lambda_1\eta_1)+\mathbb{F}_1(\xi)\exp(-U_1\Lambda_1\eta_1) + \Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{2,0}[\Lambda_1(\xi-u), U_1\Lambda_1\eta_1]\,\mathrm{d}u. \quad (52)$$

At the end of gas flow in this period, when $\eta_1 = 1$, the matrix is left at the temperature distribution

$$\theta_{s1}(\xi,1) = V_1(\Lambda_1\xi, U_1\Lambda_1) - V_{1,0}(\Lambda_1\xi, U_1\Lambda_1)+\mathbb{F}_1(\xi)\exp(-U_1\Lambda_1) + \Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{2,0}[\Lambda_1(\xi-u), U_1\Lambda_1]\,\mathrm{d}u. \quad (53)$$

In a similar way the solution (again *formal*) to the governing equations for the stronger period, equations (11) and (18), rewritten for convenience with the downstream coordinate $\zeta = 1-\xi$, i.e.

$$\frac{\partial\theta_2(\zeta,\eta_2)}{\partial\zeta}+\frac{\Lambda_1}{\sigma}[\theta_2(\zeta,\eta_2)-\theta_{s2}(\zeta,\eta_2)] = 0 \quad (54)$$

$$\frac{1}{U_1}\frac{\partial\theta_{s2}(\zeta,\eta_2)}{\partial\eta_2}+\frac{1}{\beta}\frac{\partial\theta_2(\zeta,\eta_2)}{\partial\zeta} = 0 \quad (55)$$

in $0 \leq \zeta \leq 1$ and $0 \leq \eta_2 \leq 1$, with inlet fluid temperature (see equation (22))

$$\theta_2(0,\eta_2) = 0 \quad (56)$$

and *arbitrary* initial matrix temperature distribution

$$\theta_{s2}(\zeta,0) = \mathbb{F}_2(\zeta) \quad (57)$$

is given by

$$\theta_2(\zeta,\eta_2) = \frac{\Lambda_1}{\sigma}\int_0^{\zeta}\mathbb{F}_2(v)V_{1,0}\left[\frac{\Lambda_1}{\sigma}(\zeta-v), \frac{U_1\Lambda_1}{\beta\sigma}\eta_2\right]\mathrm{d}v \quad (58)$$

and

$$\theta_{s2}(\zeta,\eta_2) = \mathbb{F}_2(\zeta)\exp\left(-\frac{U_1\Lambda_1}{\beta\sigma}\eta_2\right) + \frac{\Lambda_1}{\sigma}\int_0^{\zeta}\mathbb{F}_2(v)V_{2,0}\left[\frac{\Lambda_1}{\sigma}(\zeta-v),\frac{U_1\Lambda_1}{\beta\sigma}\eta_2\right]\mathrm{d}v. \quad (59)$$

At the end of gas flow in the stronger period, when $\eta_2 = 1$, the regenerator matrix is left with the temperature field

$$\theta_{s2}(\zeta,1) = \mathbb{F}_2(\zeta)\exp\left(-\frac{U_1\Lambda_1}{\beta\sigma}\right) + \frac{\Lambda_1}{\sigma}\int_0^{\zeta}\mathbb{F}_2(v)V_{2,0}\left[\frac{\Lambda_1}{\sigma}(\zeta-v),\frac{U_1\Lambda_1}{\beta\sigma}\right]\mathrm{d}v. \quad (60)$$

Probably the simplest way to prove that the solution given by equations (51) and (52) satisfies equations (7), (16), (21) and (50), as well as that equations (58) and (59) satisfy equations (54)–(57), is to refer properly to the fundamental properties of the Laplace transformation with respect to dimensionless time variables η_1 and η_2 taking into account the transform pairs given by equations (35) and (36).

It has been stressed that the above solutions are formal and valid for arbitrary given functions $\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$. When applied to the cyclic operation of the regenerator the reversal conditions will impose the constraints on $\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$ but the solution will remain formal until these two temperature fields are explicitly determined. Let us be more specific. The periodic equilibrium conditions for the regenerator matrix state that the spatial temperature distribution at the end of one period should coincide with that at the beginning of the other period, i.e.

$$\theta_{s1}(\xi,1) = \mathbb{F}_2(1-\xi) \quad (61)$$

and

$$\theta_{s2}(\zeta,1) = \mathbb{F}_1(1-\zeta). \quad (62)$$

When these conditions are combined with equations (53) and (60) one is left with two integral equations for $\mathbb{F}_1$ and $\mathbb{F}_2$

$$\mathbb{L}_1\{\mathbb{F}_1(\xi),\mathbb{F}_2(\xi)\} \equiv \mathbb{F}_2(1-\xi) - \Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{2,0}[\Lambda_1(\xi-u),U_1\Lambda_1]\,\mathrm{d}u - \mathbb{F}_1(\xi)\exp(-U_1\Lambda_1) - 1 + V_1(U_1\Lambda_1,\Lambda_1\xi) = 0 \quad (63)$$

$$\mathbb{L}_2\{\mathbb{F}_1(\zeta),\mathbb{F}_2(\zeta)\} \equiv \mathbb{F}_1(1-\zeta) - \frac{\Lambda_1}{\sigma}\int_0^{\zeta}\mathbb{F}_2(v)V_{2,0}\left[\frac{\Lambda_1}{\sigma}(\zeta-v),\frac{U_1\Lambda_1}{\beta\sigma}\right]\mathrm{d}v - \mathbb{F}_2(\zeta)\exp\left(-\frac{U_1\Lambda_1}{\beta\sigma}\right). \quad (64)$$

Note that the last two terms in equation (63) are the consequence of property (44).

The structure of these integral equations have made futile all the attempts to obtain $\mathbb{F}_1$ and $\mathbb{F}_2$ in a form suitable for explicit analytical evaluation of the regenerator performances. That is the main reason why any solution of the form presented above remains formal unless we are able to find even approximately, $\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$ in a closed form. The aim of this paper is to reveal that very reliable results are obtainable by solving the integral equations (63) and (64) approximately by the Galerkin method that utilizes a power series expansion as a trial solution for $\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$.

TRIAL SOLUTION

Consider the approximation of the unknown functions $\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$ in the form of the same order (M) polynomials in space variables ξ and ζ, respectively, namely let the trial solutions be

$$\tilde{\mathbb{F}}_1(\xi) = \sum_{m=0}^{M} a_{1m}\xi^m/m! \quad (65)$$

$$\tilde{\mathbb{F}}_2(\zeta) = \sum_{m=0}^{M} a_{2m}\zeta^m/m! \quad (66)$$

where a_{1m} and a_{2m} are the unknown expansion coefficients which are yet to be determined. Next suppose that these coefficients are known (they will be determined by the Galerkin method in the next section). Then, upon substitution of equations (65) and (66) into equations (51) and (52) one can obtain approximations for fluid and solid temperature fields, $\theta_1(\xi,\eta_1)$ and $\theta_{s1}(\xi,\eta_1)$, as

$$\tilde{\theta}_1(\xi,\eta_1) = V_1(\Lambda_1\xi,U_1\Lambda_1\eta_1) - \sum_{m=0}^{M}\frac{a_{1m}}{\Lambda_1^m}(-1)^m\left\{V_{1,0}(U_1\Lambda_1\eta_1,\Lambda_1\xi) + \sum_{n=1}^{m+1}(-1)^n V_n(U_1\Lambda_1\eta_1,\Lambda_1\xi)\right\} = V_1(\Lambda_1\xi,U_1\Lambda_1\eta_1) + \sum_{m=0}^{M}(-1)^m\frac{a_{1m}}{\Lambda_1^m}\sum_{n=m+2}^{\infty}(-1)^m V_n(U_1\Lambda_1\eta_1,\Lambda_1\xi) \quad (67)$$

and

$$\tilde{\theta}_{s1}(\xi,\eta_1) = 1 - V_1(U_1\Lambda_1\eta_1,\Lambda_1\xi) + \sum_{m=0}^{M}\frac{a_{1m}}{\Lambda_1^m}V_{m+1}(U_1\Lambda_1\eta_1,\Lambda_1\xi). \quad (68)$$

Here again property (44) has been used for the first two terms.

To verify equation (67) one has first to note that the integral term in equation (51) can be written as

$$\Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{1,0}[\Lambda_1(\xi-u),U_1\Lambda_1\eta_1]\,\mathrm{d}u = \Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{1,0}[U_1\Lambda_1\eta_1,\Lambda_1(\xi-u)]\,\mathrm{d}u \quad (69)$$

since the $V_{1,0}$ function is symmetric with respect to the interchange of the position of its arguments (see equation (45)), and then be regarded as a convolution for the Laplace transform

$$\mathscr{L}_{\xi\to s}\left\{\Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{1,0}[U_1\Lambda_1\eta_1,\Lambda_1(\xi-u)]\,\mathrm{d}u\right\}$$
$$=\Lambda_1\bar{\mathbb{F}}_1(s)\frac{\exp\left(-\dfrac{U_1\Lambda_1\eta_1 s}{s+\Lambda_1}\right)}{s+\Lambda_1}. \quad (70)$$

When $\mathbb{F}_1(\xi)$ is substituted by its approximation given by equation (65) one needs the inversion of

$$\Lambda_1\sum_{m=0}^{\infty}a_{1m}\frac{\exp\left(-\dfrac{U_1\Lambda_1\eta_1 s}{s+\Lambda_1}\right)}{s^{m+1}(s+\Lambda_1)} \quad (71)$$

in order to calculate the integral term (69). Since, for $\mathrm{Re}(s)>\Lambda_1$

$$\frac{1}{s^{m+1}(s+\Lambda_1)}=\left(\frac{-1}{\Lambda_1}\right)^{m+1}\left[\frac{1}{s+\Lambda_1}+\frac{1}{\Lambda_1}\sum_{n=1}^{m+1}\left(-\frac{\Lambda_1}{s}\right)^n\right]$$
$$=\frac{(-1)^m}{\Lambda_1^{m+2}}\sum_{n=m+2}^{\infty}\left(-\frac{\Lambda_1}{s}\right)^n \quad (72)$$

term by term inversion $s\to\xi$ of (71), recalling that equations (35) and (36) hold, will yield the result as given by equation (67), confirming thus relations (38) and (47).

The Laplace transform of the convolution term in equation (62), using equation (43), is

$$\mathscr{L}_{\xi\to s}\left\{\Lambda_1\int_0^{\xi}\mathbb{F}_1(u)V_{2,0}[\Lambda_1(\xi-u),U_1\Lambda_1\eta_1]\,\mathrm{d}u\right\}$$
$$=\bar{\mathbb{F}}_1(s)\left[\exp\left(-\frac{U_1\Lambda_1\eta_1 s}{s+\Lambda_1}\right)-\exp(-U_1\Lambda_1\eta_1)\right] \quad (73)$$

so that, for $\mathbb{F}_1(\xi)$ approximated as in equation (65), the straightforward inversion of

$$\sum_{m=0}^{M}\frac{a_{1m}}{\Lambda^{m+1}}\left[\exp\left(-\frac{U_1\Lambda_1\eta_1 s}{s+\Lambda_1}\right)-\exp(-U_1\Lambda_1\eta_1)\right]\Big/(s/\Lambda_1)^{m+1} \quad (74)$$

confirms the result given by equation (68).

In a similar way it can be shown that the approximations for fluid and solid temperature fields $\theta_2(\zeta,\eta_2)$ and $\theta_{s2}(\zeta,\eta_2)$ given by equations (58) and (59), respectively, with $\mathbb{F}_2(\zeta)$ of the form given by equation (66), are

$$\tilde{\theta}_2(\zeta,\eta_2)=-\sum_{m=0}^{M}(-1)^m\frac{a_{2m}}{(\Lambda_1/\sigma)^m}$$
$$\times\left[V_{1,0}\left(\frac{U_1\Lambda_1}{\beta\sigma}\eta_2,\frac{\Lambda_1}{\sigma}\zeta\right)+\sum_{n=1}^{m+1}(-1)^n\right.$$
$$\left.\times V_n\left(\frac{U_1\Lambda_1}{\beta\sigma}\eta_2,\frac{\Lambda_1}{\sigma}\zeta\right)\right]=\sum_{m=0}^{M}(-1)^m\frac{a_{2m}}{(\Lambda_1/\sigma)^m}$$
$$\times\sum_{n=m+2}^{\infty}(-1)^nV_n\left(\frac{U_1\Lambda_1}{\beta\sigma}\eta_2,\frac{\Lambda_1}{\sigma}\zeta\right) \quad (75)$$

and

$$\tilde{\theta}_{s2}(\zeta,\eta_2)=\sum_{m=0}^{M}\frac{a_{2m}}{(\Lambda_1/\sigma)^m}V_{m+1}\left(\frac{U_1\Lambda_1}{\beta\sigma}\eta_2,\frac{\Lambda_1}{\sigma}\zeta\right). \quad (76)$$

So far it has been demonstrated that the temperature fields in either period of the regenerator operation can be obtained in a closed form if the matrix temperature distributions at the start of each period are of the arbitrary polynomial order M in the spatial coordinate. However, the main advantage of using equations (65) and (66) as the trial solutions, is in evaluating the regenerator effectiveness. Namely, from various equivalent possibilities to find ε from a known solution, as discussed above, formula (34) is the most suitable and can be rewritten, upon using equations (61) and (62), as

$$\varepsilon=\frac{1}{U_1}\int_0^1[\theta_{s1}(\xi,1)-\theta_{s2}(\xi,1)]\,\mathrm{d}\xi$$
$$=\frac{1}{U_1}\int_0^1[\mathbb{F}_2(1-\xi)-\mathbb{F}_1(\xi)]\,\mathrm{d}\xi$$
$$=\frac{1}{U_1}\left[\int_0^1\mathbb{F}_2(\zeta)\,\mathrm{d}\zeta-\int_0^1\mathbb{F}_1(\xi)\,\mathrm{d}\xi\right]. \quad (77)$$

Then, with the trial solution of the form of equations (65) and (66), the effectiveness is given by

$$\varepsilon=\frac{1}{U_1}\sum_{m=0}^{M}\frac{a_{2m}-a_{1m}}{(m+1)!} \quad (78)$$

and the problem is just to determine the coefficients a_{1m} and a_{2m} for $m=0,1,2,\ldots,M$. For any fixed M one needs $2(M+1)$ equations for $M+1$ unknown a_{1m} and for $M+1$ unknown a_{2m} coefficients. These equations are derived in the next section using the Galerkin method and it is shown that the elements of the matrix of a set of $2(M+1)$ linear algebraic equations are obtainable explicitly in terms of regenerator parameters Λ_1, U_1, σ and β.

THE GALERKIN METHOD

Introducing the trial solutions $\tilde{\mathbb{F}}_1(\xi)$ and $\tilde{\mathbb{F}}_2(\zeta)$ into the integral equations (63) and (64), one finds the residuals

$$\mathscr{R}_1(\xi)\equiv\mathbb{L}_1\{\tilde{\mathbb{F}}_1(\xi),\tilde{\mathbb{F}}_2(\xi)\} \quad (79)$$

and

$$\mathscr{R}_2(\zeta)\equiv\mathbb{L}_2\{\tilde{\mathbb{F}}_1(\zeta),\tilde{\mathbb{F}}_2(\zeta)\} \quad (80)$$

in $0\leqslant\zeta,\xi\leqslant 1$. With the trial solutions of the form of equations (65) and (66) the Galerkin method can be applied to the integral equations (63) and (64) as

$$\int_0^1 \mathscr{R}_1(\xi)\frac{\xi^k}{k!}\mathrm{d}\xi = 0, \quad k = 0,1,2,\ldots,M \quad (81)$$

and

$$\int_0^1 \mathscr{R}_2(\zeta)\frac{\zeta^k}{k!}\mathrm{d}\zeta = 0, \quad k = 0,1,2,\ldots,M \quad (82)$$

which yields $2(M+1)$ algebraic equations for the determination of the expansion coefficients a_{1m} and a_{2m} for each m. Since the trial solutions, equations (65) and (66), when introduced into integral equations (63) and (64), give the residuals of the form

$$\mathscr{R}_1(\xi) = \sum_{m=0}^{M} a_{2m}\frac{(1-\xi)^m}{m!} - \sum_{m=0}^{M}\frac{a_{1m}}{\Lambda_1^m}V_{m+1}(U_1\Lambda_1,\Lambda_1\xi) - 1 + V_1(U_1\Lambda_1,\Lambda_1\xi) \quad (83)$$

and

$$\mathscr{R}_2(\zeta) = \sum_{m=0}^{M} a_{1m}\frac{(1-\zeta)^m}{m!} - \sum_{m=0}^{M}\frac{a_{2m}}{(\Lambda_1/\sigma)^m}V_{m+1}\left(\frac{U_1\Lambda_1}{\beta\sigma},\frac{\Lambda_1}{\sigma}\zeta\right) \quad (84)$$

respectively, the resulting set of algebraic equations (81) and (82) can be written as

$$\sum_{m=0}^{M}[-A_{mk}(\Pi_1,\Lambda_1)a_{1m}+B_{mk}a_{2m}] = C_k, \quad k = 0,1,2,\ldots,M \quad (85)$$

$$\sum_{m=0}^{M}[B_{mk}a_{1m}-A_{mk}(\Pi_2,\Lambda_2)a_{2m}] = 0, \quad k = 0,1,2,\ldots,M \quad (86)$$

where

$$A_{mk}(\Pi_j,\Lambda_j) \equiv \frac{1}{\Lambda_j^m}\int_0^1 V_{m+1}(\Pi_j,\Lambda_j\xi)\frac{\xi^k}{k!}\mathrm{d}\xi = \sum_{i=0}^{M}\frac{(-1)^i}{(k-i)!}V_{i+m+2}(\Pi_j,\Lambda_j)/\Lambda_j^{i+m+1}, \quad j = 1,2 \quad (87)$$

$$B_{mk} \equiv \int_0^1 \frac{(1-x)^m}{m!}\frac{x^k}{k!}\mathrm{d}x = \frac{1}{(m+k+1)!} \quad (88)$$

$$C_k \equiv \int_0^1 [1-V_1(\Pi_1,\Lambda_1\xi)]\frac{\xi^k}{k!}\mathrm{d}\xi = \frac{1}{(k+1)!} - \sum_{i=0}^{k}\frac{(-1)^i}{(k-i)!}\frac{V_{i+2}(\Pi_1,\Lambda_1)}{\Lambda_1^{i+1}}. \quad (89)$$

Equations (87) and (89) are the consequences of property (49). Note that $C_k = B_{0k}-A_{0k}$, and that coefficients B_{mk} are independent of regenerator parameters and are exactly the same in both equations (85) and (86). The functional form of the A_{mk} dependence of the regenerator parameters is the same as well, but in equation (85) A_{mk} depend on $\Pi_1 = U_1\Lambda_1$ and Λ_1, while in equation (86) A_{mk} depend on $\Pi_2 = U_1\Lambda_1/\beta\sigma$ and $\Lambda_2 = \Lambda_1/\sigma$.

Once the expansion coefficients a_{1m}, a_{2m} ($m = 0,1,2,\ldots,M$) are determined, for specified Λ_1, U_1, σ and β (or Λ_1, Π_1, Λ_2 and Π_2), from equations (85) and (86), the temperature fields of either fluid or regenerator matrix at any position and any time instance are readily obtainable from equations (67), (68), (75) and (76). However, the most straightforward result to be obtained from known values of a_{1m} and a_{2m} is the regenerator effectiveness given by equation (78).

When very precise results are required one must use higher approximations by increasing the order M of the trial polynomials. This will confirm the known feature of the Galerkin method: in the limit $M \to \infty$ it forces the residuals to be zero by making them (see equations (81) and (82)) orthogonal to each linearly independent member of the complete set of trial functions $\xi^k/k!$ ($k = 0,1,2,\ldots,M$).

For design purposes one is primarily interested in reliable results for regenerator effectiveness and has to be assured that by increasing M (i.e. reducing the values of the residuals in the regenerator space $0 \leqslant \xi \leqslant 1$), the results for ε correspond to an exact solution to certain decimal places. We now illustrate the convergence of this method by carrying out the computations to higher order terms.

The coefficients a_{1m} and a_{2m} ($m = 0,1,2,\ldots,M$) obtained from the algebraic set of equations (85) and (86), as well as the corresponding values of regenerator effectiveness are presented in Table 1 for values of M up to 5 in the case of, arbitrarily chosen, reduced lengths $\Lambda_1 = 15.5$, $\Lambda_2 = 18$, reduced period $\Pi_2 = 16$ and larger reduced length to period ratio $(\Lambda/\Pi)_1 = 1.2$. It is evident from Table 1 that the regenerator effectiveness is at the practically correct value (three significant figures) already at $M = 2$, and that the corresponding effectiveness results for $M = 4$ and 5 coincide for six decimal places. The latter fact is true for the great majority of different combinations of four regenerator parameter values, and not just for the example presented in Table 1.

The convergence towards the exact solution can be seen also by following the development of matrix temperature distributions at the end of respective periods ($\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$ given by equations (65) and (66), respectively) with increasing the order (M) of the trial solution. This is presented in Fig 2 for the same values of parameters as in Table 1. Obviously constant matrix temperatures ($M = 0$) and linear distributions ($M = 1$) are very rough and unrealistic approximations. However, starting already with the second-order polynomials ($M = 2$), correct shapes of the temperature distributions are established, and they are almost indistinguishable from the plots of the distributions for $M = 3$, 4 and 5. Table 2 provides the numerical values of $\mathbb{F}_1$ and $\mathbb{F}_2$ for $M = 3$, 4 and 5, wherefrom it becomes clear that the solution has practically converged to the exact one at $M = 5$.

We end this section by concluding that the Galerkin method provides practically accurate results for the regenerator effectiveness with the second-order

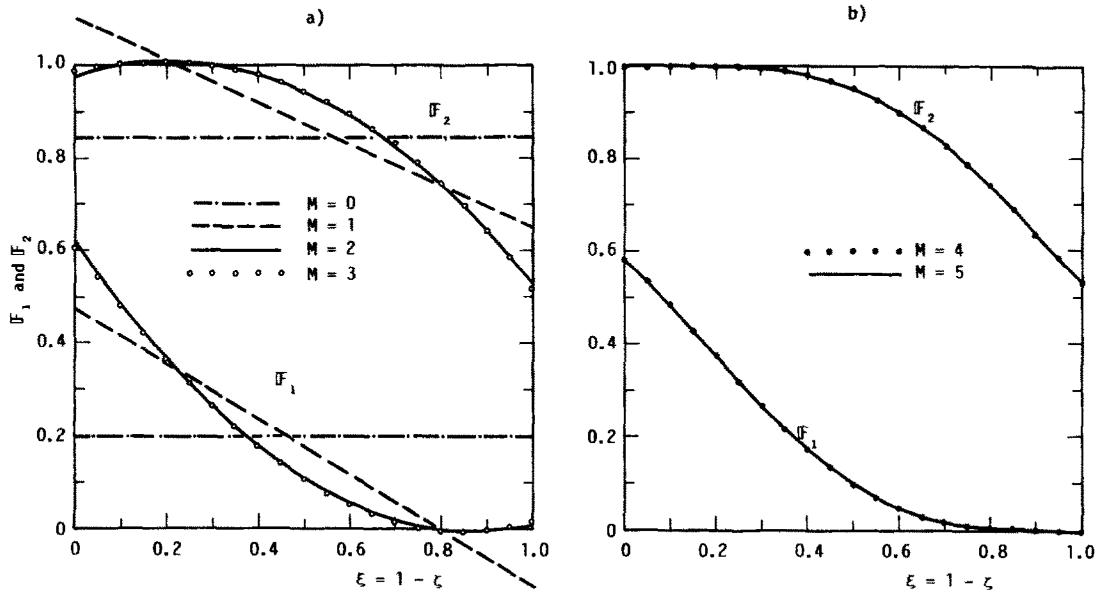

FIG. 2. Matrix temperature distributions $\mathbb{F}_1(\xi)$ and $\mathbb{F}_2(\zeta)$ for various orders (M) of the trial solution: (a) $M = 0, 1, 2$ and 3; (b) $M = 4$ and 5 ($\Lambda_1 = 15.5$, $U_1 = 0.8333$, $\beta = 0.9375$, $\sigma = 0.8611$).

Table 1. Convergence of ε given by equation (78) by increasing the order (M) of the trial polynomials for $\Lambda_1 = 15.5$, $\Lambda_2 = 18$, $(\Lambda/\Pi)_1 = 1.2$ and $\Pi_2 = 16$ ($U_1 = 0.8333$, $\Lambda_1 = 15.5$, $\beta = 0.9375$, $\sigma = 0.8611$)

M	Expansion coefficients from equations (85) and (86) $a_{1m}/(m+1)!$	$a_{2m}/(m+1)!$	$\varepsilon = (\Lambda_1/\Pi_1)\Sigma_{m=0}^{M}(a_{2m}-a_{1m})/(m+1)!$
0	$a_{10} = 0.1994847655E+00$	$a_{20} = 0.8464496251E+00$	0.776358
1	$a_{10} = 0.4774757624E+00$ $a_{11}/2! = -0.3004407361E+00$	$a_{20} = 0.6495968994E+00$ $a_{21}/2! = 0.2269321763E+00$	0.839393
2	$a_{10} = 0.6123050451E+00$ $a_{11}/2! = -0.7089718580E+00$ $a_{12}/3! = 0.2714202073E+00$	$a_{20} = 0.5296996142E+00$ $a_{21}/2! = 0.6069868991E+00$ $a_{22}/3! = -0.2558692859E+00$	0.847277
3	$a_{10} = 0.6038717628E+00$ $a_{11}/2! = -0.6536371708E+00$ $a_{12}/3! = 0.1770038495E+00$ $a_{13}/4! = 0.4766383761E-01$	$a_{20} = 0.5172719219E+00$ $a_{21}/2! = 0.6850765545E+00$ $a_{22}/3! = -0.3877162736E+00$ $a_{23}/4! = 0.6629797653E-01$	0.847233
4	$a_{10} = 0.5845412612E+00$ $a_{11}/2! = -0.4595721960E+00$ $a_{12}/3! = -0.4069373320E+00$ $a_{13}/4! = 0.7302788247E+00$ $a_{14}/5! = -0.2734096322E+00$	$a_{20} = 0.5321021858E+00$ $a_{21}/2! = 0.5390921266E+00$ $a_{22}/3! = 0.5029928890E-01$ $a_{23}/4! = -0.4455804284E+00$ $a_{24}/5! = 0.2050801859E+00$	0.847311
5	$a_{10} = 0.5850397944E+00$ $a_{11}/2! = -0.4669225216E+00$ $a_{12}/3! = -0.3711900982E+00$ $a_{13}/4! = 0.6554213580E+00$ $a_{14}/5! = -0.2031960408E+00$ $a_{15}/6! = -0.2424905086E-01$	$a_{20} = 0.5351881492E+00$ $a_{21}/2! = 0.4929495210E+00$ $a_{22}/3! = 0.2655371695E+00$ $a_{23}/4! = -0.8760081957E+00$ $a_{24}/5! = 0.5924537939E+00$ $a_{25}/6! = -0.1291244293E+00$	0.847311

($M = 2$), and very precise results with fifth-order ($M = 5$) trial solution.

RESULTS

The Galerkin method based computer software has been designed for an arbitrary combination of four regenerator parameters and successfully utilized on a variety of computing facilities. Here some sample results obtained on the Amdahl V/7 computer are presented.

The counterflow regenerator effectiveness has been simulated with $M = 5$ for the following range of parameters $1 \leqslant \Lambda_1 \leqslant 1000$, $0 \leqslant \Pi_1 \leqslant 2000$, $0.2 \leqslant \Lambda_2 \leqslant 2000$ and $0 \leqslant \Pi_2 \leqslant 10\,000$ which covers a wide range of unbalance and asymmetry factors. To cover this range in just 25 tables (or charts) ε is presented as a function of $U_1 \in [0, 2]$ using Λ_1 as a par-

Table 2. Convergence of $\mathbb{F}_1$ and $\mathbb{F}_2$ given by equations (65) and (66), respectively, by increasing the order (M) of the trial polynomials for the same values of parameters as in Table 1

	$\mathbb{F}_1$			$\mathbb{F}_2$		
$\xi = 1-\zeta$	$M = 3$	$M = 4$	$M = 5$	$M = 3$	$M = 4$	$M = 5$
0.00	0.6039	0.5845	0.5850	0.9895	1.0043	1.0012
0.05	0.5399	0.5359	0.5359	0.9965	0.9996	0.9998
0.10	0.4786	0.4832	0.4830	1.0016	0.9981	0.9994
0.15	0.4204	0.4284	0.4282	1.0044	0.9983	0.9994
0.20	0.3652	0.3731	0.3730	1.0048	0.9987	0.9992
0.25	0.3132	0.3188	0.3188	1.0025	0.9981	0.9979
0.30	0.2646	0.2667	0.2669	0.9974	0.9956	0.9948
0.35	0.2195	0.2180	0.2182	0.9893	0.9902	0.9892
0.40	0.1781	0.1735	0.1736	0.9779	0.9812	0.9803
0.45	0.1405	0.1338	0.1339	0.9631	0.9680	0.9675
0.50	0.1068	0.0995	0.0994	0.9447	0.9502	0.9502
0.55	0.0772	0.0706	0.0705	0.9225	0.9275	0.9280
0.60	0.0519	0.0474	0.0471	0.8962	0.8997	0.9006
0.65	0.0309	0.0295	0.0293	0.8657	0.8669	0.8680
0.70	0.0144	0.0167	0.0165	0.8308	0.8293	0.8302
0.75	0.0025	0.0083	0.0083	0.7913	0.7872	0.7875
0.80	−0.0045	0.0036	0.0038	0.7469	0.7412	0.7407
0.85	−0.0066	0.0016	0.0019	0.6975	0.6917	0.6906
0.90	−0.0036	0.0010	0.0013	0.6429	0.6397	0.6385
0.95	0.0047	0.0006	0.0006	0.5829	0.5862	0.5861
1.00	0.0183	−0.0013	−0.0021	0.5173	0.5321	0.5352

	$\beta = U_1/U_2$				
$\sigma\beta$	1.0	0.8	0.6	0.4	0.2
1.0	σ= 1	σ=5/4	σ=5/3	σ=5/2	σ= 5
0.8	σ=4/5	σ= 1	σ=4/3	σ=4/2	σ= 4
0.6	σ=3/5	σ=3/4	σ= 1	σ=3/2	σ= 3
0.4	σ=2/5	σ=2/4	σ=2/3	σ= 1	σ= 2
0.2	σ=1/5	σ=1/4	σ=1/3	σ=1/2	σ= 1

BALANCED UNBALANCED

ASYMMETRIC SYMMETRIC ASYMMETRIC

FIG. 3. Survey of parameter values that are covered in 25 regenerator effectiveness charts.

ameter for fixed β and σ. To provide an accurate linear interpolation within a table as well as among the tables, the following twenty Λ_1 values have been selected: 1, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7, 8.5, 10, 12, 15, 18, 23, 30, 50, 100 and 1000, and each effectiveness table (or chart) was for one of five values of β and $\sigma\beta$ from Fig. 3. A typically generated effectiveness table is presented in Table 3, and four typical effectiveness charts are presented in Figs. 4–7. The complete set of charts will be published elsewhere [24].

We note that all five figures of effectiveness values in Table 3 are accurate. To the best of the authors' knowledge there is no method reported that gives as accurate results in such a wide range of parameters. Also it is worth noting that an average CPU time for generating a table like the one presented in Table 3 was 90 s on the Amdahl V/7 computer.

The results presented in Figs. 4–7 correspond to the combination of parameters from the four corners of Fig. 3, but arranged in a sequence of increasing favorableness of counterflow regenerator performance. The lowest effectiveness values are those in Fig. 4 for a symmetric ($\sigma = 1$) and balanced ($\beta = 1$) regenerator. The balanced ($\beta = 1$) and highly asymmetric ($\sigma = 0.2$)

Table 3. Counterflow regenerator effectiveness with 20% unbalance ($\beta = 0.8$) and 50% asymmetry ($\sigma = 0.5$)

$\Lambda_1 \rightarrow$	1.0	1.5	2.0	2.5	3.0	3.5	4	5	6	7
0.10	0.43406	0.54390	0.62245	0.68128	0.72690	0.76322	0.79276	0.83771	0.87006	0.89426
0.20	0.43324	0.54241	0.62035	0.67867	0.72387	0.75986	0.78913	0.83372	0.86587	0.88998
0.30	0.43188	0.53994	0.61688	0.67437	0.71890	0.75436	0.78323	0.82728	0.85918	0.88324
0.40	0.43000	0.53652	0.61209	0.66845	0.71208	0.74684	0.77518	0.81858	0.85020	0.87422
0.50	0.42760	0.53218	0.60603	0.66098	0.70350	0.73740	0.76510	0.80771	0.83897	0.86291
0.60	0.42471	0.52697	0.59878	0.65205	0.69325	0.72614	0.75307	0.79467	0.82543	0.84918
0.70	0.42134	0.52094	0.59040	0.64176	0.68144	0.71313	0.73913	0.77945	0.80946	0.83279
0.80	0.41753	0.51415	0.58100	0.63021	0.66816	0.69847	0.72336	0.76204	0.79096	0.81356
0.90	0.41329	0.50667	0.57067	0.61753	0.65356	0.68229	0.70586	0.74251	0.76995	0.79144
1.00	0.40867	0.49856	0.55952	0.60385	0.63778	0.66475	0.68682	0.72106	0.74661	0.76660
1.10	0.40370	0.48990	0.54767	0.58933	0.62101	0.64607	0.66649	0.69798	0.72132	0.73945
1.20	0.39840	0.48077	0.53524	0.57413	0.60347	0.62651	0.64517	0.67368	0.69457	0.71063
1.30	0.39282	0.47125	0.52236	0.55843	0.58537	0.60634	0.62318	0.64860	0.66696	0.68086
1.40	0.38700	0.46142	0.50914	0.54239	0.56693	0.58584	0.60086	0.62322	0.63907	0.65086
1.50	0.38096	0.45136	0.49572	0.52618	0.54838	0.56527	0.57853	0.59796	0.61143	0.62127
1.60	0.37474	0.44114	0.48221	0.50996	0.52990	0.54486	0.55646	0.57317	0.58449	0.59258
1.70	0.36838	0.43083	0.46870	0.49386	0.51166	0.52483	0.53490	0.54914	0.55855	0.56513
1.80	0.36192	0.42049	0.45528	0.47800	0.49381	0.50533	0.51402	0.52606	0.53383	0.53912
1.90	0.35537	0.41018	0.44205	0.46248	0.47646	0.48649	0.49394	0.50406	0.51042	0.51466
2.00	0.34878	0.39995	0.42906	0.44737	0.45969	0.46838	0.47474	0.48321	0.48839	0.49175
$\uparrow$ U_1 $\downarrow$										
0.10	0.92061	0.93922	0.95651	0.97285	0.98265	0.99153	0.99687	1.00000	1.00000	1.00000
0.20	0.91636	0.93510	0.95269	0.96957	0.97993	0.98964	0.99576	0.99983	1.00000	1.00000
0.30	0.90975	0.92880	0.94691	0.96466	0.97584	0.98671	0.99397	0.99924	1.00000	1.00000
0.40	0.90095	0.92040	0.93917	0.95799	0.97019	0.98252	0.99132	0.99860	1.00000	1.00000
0.50	0.88985	0.90972	0.92919	0.94916	0.96249	0.97652	0.98719	0.99736	0.99994	1.00000
0.60	0.87619	0.89636	0.91645	0.93752	0.95199	0.96782	0.98064	0.99467	0.99955	1.00000
0.70	0.85958	0.87984	0.90030	0.92220	0.93766	0.95518	0.97023	0.98899	0.99855	1.00000
0.80	0.83970	0.85965	0.88003	0.90223	0.91824	0.93693	0.95379	0.97726	0.99390	0.99998
0.90	0.81637	0.83550	0.85515	0.87676	0.89257	0.91138	0.92887	0.95518	0.97846	0.99984
1.00	0.78974	0.80746	0.82566	0.84567	0.86031	0.87778	0.89415	0.91932	0.94351	0.98216
1.10	0.76030	0.77613	0.79225	0.80976	0.82242	0.83731	0.85101	0.87146	0.88998	0.90888
1.20	0.72886	0.74250	0.75617	0.77071	0.78097	0.79271	0.80310	0.81751	0.82829	0.83333
1.30	0.69637	0.70775	0.71890	0.73045	0.73833	0.74701	0.75427	0.76324	0.76820	0.76923
1.40	0.66375	0.67298	0.68181	0.69065	0.69645	0.70253	0.70726	0.71225	0.71412	0.71429
1.50	0.63177	0.63911	0.64592	0.65249	0.65660	0.66066	0.66353	0.66604	0.66665	0.66667
1.60	0.60101	0.60673	0.61188	0.61663	0.61944	0.62203	0.62367	0.62483	0.62500	0.62500
1.70	0.57181	0.57620	0.58002	0.58337	0.58524	0.58682	0.58770	0.58819	0.58824	0.58824
1.80	0.54436	0.54768	0.55047	0.55278	0.55397	0.55490	0.55535	0.55554	0.55556	0.55556
1.90	0.51872	0.52121	0.52320	0.52475	0.52550	0.52602	0.52624	0.52631	0.52632	0.52632
2.00	0.49487	0.49671	0.49811	0.49914	0.49959	0.49987	0.49997	0.50000	0.50000	0.50000
$\Lambda_1 \rightarrow$	8.5	10	12	15	18	23	30	50	100	1000

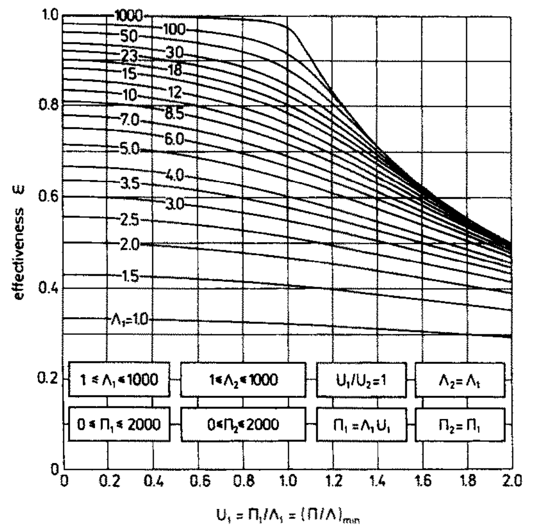

FIG. 4. Counterflow regenerator effectiveness for $\beta = 1$ and $\sigma = 1$.

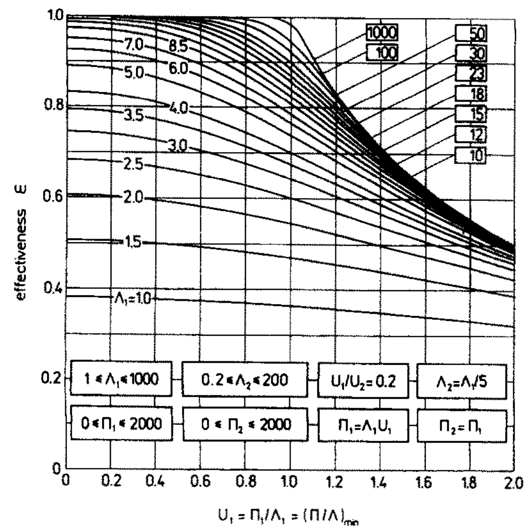

FIG. 6. Counterflow regenerator effectiveness for $\beta = 0.2$ and $\sigma = 5$.

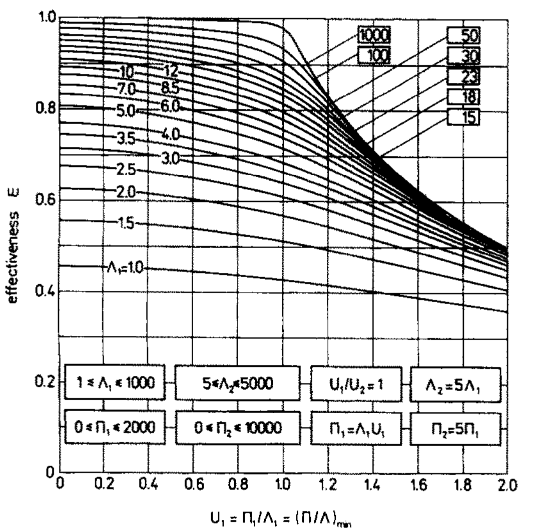

FIG. 5. Counterflow regenerator effectiveness for $\beta = 1$ and $\sigma = 0.2$.

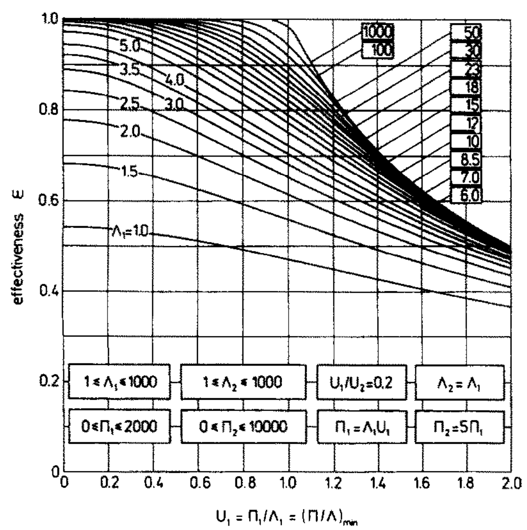

FIG. 7. Counterflow regenerator effectiveness for $\beta = 0.2$ and $\sigma = 1$.

regenerator is more favorable for all Λ_1 values as can be seen from Fig. 5.

Figure 6 presents the results for a highly unbalanced ($\beta = 0.2$) and highly asymmetric ($\sigma = 5$) regenerator. Except for low values of Λ_1 ($\Lambda_1 \leqslant 2.5$) this is a much better regenerator performance than the case presented in Fig. 5.

Among the four cases presented the highest effectiveness values for all Λ_1 values are attained at highly unbalanced ($\beta = 0.2$) but symmetric ($\sigma = 1$) operation of the regenerator as shown in Fig. 7.

CONCLUDING REMARKS

The present method provides a very simple and straightforward solution to the unbalanced and asymmetric counterflow regenerator problem for any arbitrary combination of the four regenerator parameters. Compared to various other approximate methods the present solution has no computational restriction associated with the large values of Π and Λ. The convergence of the solution is found to be very fast and the computation time very short which made it possible to investigate regenerator effectiveness in an extremely wide range of parameters and readily to generate appropriate tabulations and effectiveness charts. Sample results of the analysis are presented in Table 3 and Figs. 4–7. The use of the Galerkin method has proved to be very powerful for solving the set of integral equations associated with the general counterflow regenerator problem. Selection of alternative sets of four regenerator parameters (U_1, Λ_1, β and σ)

made it possible to provide a physically meaningful interpretation of regenerator performances.

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PROBLEME DU REGENERATEUR THERMIQUE A CONTRE-COURANT ET BILAN ASYMETRIQUE: SOLUTION PAR LA METHODE DE GALERKIN ET SIGNIFICATION DES PARAMETRES ADIMENSIONNELS

Résumé—Le problème du régénérateur thermique à contre-courant et bilan asymétrique, décrit par les idéalisations classiques est résolu par la méthode de Galerkin. Les équations intégrales relatives aux conditions à l'équilibre cyclique de la matrice du régénérateur sont transformées et un système d'équations algébriques. Ceci permet la détermination des coefficients de développement associés à la représentation des distributions de température de matrice au départ de chaque période du cycle sous la forme d'une série puissance en fonction de la variable d'espace. La méthode est aisée et d'application directe et elle conduit à des expressions analytiques explicites des coefficients du développement pour une combinaison quelconque des quatre paramètres sans dimension du régénérateur. Un accord excellent est trouvé entre les résultats de cette solution nouvelle et ceux déjà connus par différentes solutions numériques. On discute la convergence entre les calculs numériques exacts. La solution est utilisée pour prédire l'efficacité dans le cas d'un large domaine pour les quatre paramètres sans dimension. On présente les raisons thermodynamiques, d'une façon alternative mais rationnelle et chargée de sens, de définir les quatre paramètres du régénérateur.

BERECHNUNG VON GEGENSTROM-REGENERATOREN MIT HILFE DES GALERKIN-VERFAHRENS UND DIE BEDEUTUNG DIMENSIONSLOSER PARAMETER

Zusammenfassung—Der thermische Gegenstrom-Regenerator wird unter Verwendung der klassischen Idealisierung beschrieben und mittels der Galerkin-Methode berechnet. Die Integralgleichungen zur Beschreibung der zyklisch wiederkehrenden Vorgänge in der Regenerator-Matrix werden in einen Satz algebraischer Gleichungen transformiert. Dies erlaubt die Bestimmung der Koeffizienten bei der Beschreibung der Temperaturverteilung in der Matrix zu Beginn eines Zyklus in Form einer Potenzreihe in Abhängigkeit von der Raumvariablen. Das Verfahren ist leicht und in einem Zuge anwendbar und führt zu expliziten analytischen Ausdrücken für die Koeffizienten der Reihenentwicklung bei beliebiger Kombination der vier dimensionslosen Parameter des Regenerators. Die Übereinstimmung der Ergebnisse aus dieser neuen Lösung mit denjenigen, welche in der Literatur aufgrund unterschiedlicher numerischer Berechnungen angegeben werden, ist hervorragend. Die Annäherung—bei Berechnungen unter Verwendung von Ausdrücken höherer Ordnung—an die exakte Lösung wird diskutiert. Die Lösung wird für Wirkungsgradberechnungen in einem weiten Bereich der vier dimensionslosen Parameter verwendet. Es werden thermodynamische Gründe für eine abgewandelte, jedoch sinnvolle Art der Definition der vier Regenerator-Parameter vorgestellt.

ЗАДАЧА АСИММЕТРИЧНО-НЕСБАЛАНСИРОВАННОГО ПРОТИВОТОЧНОГО ТЕПЛОВОГО РЕГЕНЕРАТОРА: РЕШЕНИЕ МЕТОДОМ ГАЛЕРКИНА И ЗНАЧЕНИЕ БЕЗРАЗМЕРНЫХ ПАРАМЕТРОВ

Аннотация—Методом Галеркина решается задача асимметрично-несбалансированного противоточного теплового регенератора, описываемая классическими идеализациями. Интегральные уравнения, относящиеся к условиям обращения при цилиндрическом равновесии матрицы регенератора, преобразуются в систему алгебраических уравнений. Это позволяет определить коэффициенты разложения посредством представления матрицы распределений температур в начале каждого периода цикла в виде степенного ряда через пространственную переменную. Предложенный метод является простым и легко применимым и позволяет получить явные аналитические выражения для коэффициента разложения при любой комбинации четырех безразмерных параметров асимметрично-несбалансированного регенератора. Получено хорошее совпадение результатов представленного нового рещения и результатов различных численых решений, имеющихся в литературе. Обсуждается приближение результатов к точным при расчетах членов более высокого порядка. Полученное решение используется для определения эффективности широкого диапазона изменений четырех безразмерных параметров. Представлено термодинамическое обоснование альтернативного, но рационального и целесообразного способа определения четырех параметров регенератора.